Mathematical Foundations of Neuroscience - Sample Questions - Lecture 5 - 2d systems

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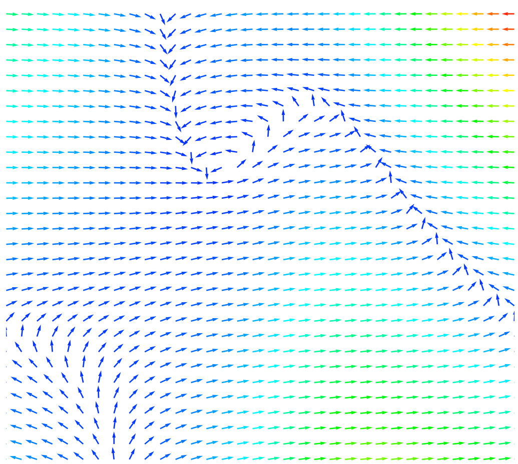
Questions marked with * are not obligatory.

1. Show than any n-dimensional non-autonomous ODE can be expressed as $n+1$ dimensional autonomous ODE. Show that any 1 dimensional n'th-order ODE can be expressed as first order n dimensional ODE.

2. Given the vector field below, try to sketch its nullclines and some trajectories. Can you spot any equilibrium points?



3. Find the eigenvalues of the matrix:

$$A = \begin{bmatrix} 3 & 2 \\ 6 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ -5 & 2 \end{bmatrix}$$

4. Find the eigenvectors of the above matrices.

5. Determine the nullclines and draw the phase portrait of the system

$$\frac{dx}{dt} = x - x^3/3 - y$$
$$\frac{dy}{dt} = by$$

where $b > 0$ is a parameter.

6. Show that

$$\begin{bmatrix} v(x) \\ w(y) \end{bmatrix} = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} v_2$$

is the solution of 2d linear ODE $\frac{d\overrightarrow{x}}{dt} = A\overrightarrow{x}$, where λ_1 and λ_2 are eigenvalues of matrix A and v_1, v_2 are the corresponding eigenvectors.

7. Characterize the possible hyperbolic equilibria in 2d system.

8. How many equilibrium points can a system

$$\frac{dx}{dt} = W_n(x) + y$$
$$\frac{dy}{dt} = x - y$$

have, where W_n is an n-th degree polynomial. *