

# Mathematical Foundations of Neuroscience - Sample Questions - Lecture 6 - Bifurcations in 2d

Filip Piękniewski

November 9, 2009

Questions marked with \* are not obligatory.

1. Describe all bifurcations of equilibrium that lead from stability to spiking. Discuss any characteristic features these bifurcations have.
2. Describe all bifurcations of limit cycles that lead from spiking to resting. Discuss any characteristic features these bifurcations have.
3. Describe how to estimate frequency of spiking in systems undergoing saddle node on invariant circle bifurcation.
4. Prove that the system

$$\begin{aligned}\frac{dv}{dt} &= I + v^2 - u \\ \frac{du}{dt} &= a(bv - u)\end{aligned}$$

with  $a > 0$  undergoes

- saddle-node bifurcation when  $b^2 = 4I$
  - Andronov-Hopf bifurcation when  $a < b$  and  $a^2 - 2ab + 4I = 0$
5. (\*) Show that the non-degeneracy and transversality conditions are necessary for the Andronov-Hopf bifurcation. That is, present a system that does not exhibit Andronov-Hopf bifurcation, but satisfies
    - the non-hyperbolicity and non-degeneracy conditions, or
    - the non-hyperbolicity and transversality conditions.