Persistent activation blobs in spiking neural networks with mexican hat connectivity

Filip Piekniewski⋆

Faculty of Mathematics and Computer Science,
Nicolaus Copernicus University, 87-100 Torun, Poland
`philip@mat.umk.pl`, `http://www.mat.umk.pl/~philip/`

Abstract. Short range excitation, long range inhibition sometimes referred to as mexican hat connectivity seems to play important role in organization of the cortex, leading to fairly well delineated sites of activation. In this paper we study a computational model of a grid filled with rather simple spiking neurons with mexican hat connectivity. The simulation shows, that when stimulated with small amount of random noise, the model results in a stable activated state in which the spikes are organized into persistent blobs of activity. Furthermore, these blobs exhibit significant lifetime, and stable movement across the domain. We analyze lifetimes and trajectories of the spots, arguing that they can be interpreted as basic computational *charge* units of the so called *spike flow model* introduced in earlier work.

Key words: spiking networks, mexican hat connectivity, spike flow model

1 Introduction

It is a subject of ongoing discussion of what is the elementary computational unit of the brain, whether this important role should be attributed to a neuron and an action potential, or rather a group of neurons, possibly a polychronous group of spikes [1] or maybe some bigger ensemble with more complex dynamics (microcolumn etc.). In our previous work [2,3,4,5] we studied the so called *spike flow model* in which neuron-like units exchange quants of some persistent charge (that is conserved by the dynamics), leading eventually to a winner-take-all dynamics and scale-free (power law) connectivity of the resulting charge transfer graph. The study was motivated by experimental results, which showed that functional brain networks have certain connectivity properties [6,7]. The model, though leading to similar properties of the charge exchange graph (power law degree distribution with exponent $\gamma = 2$), lacked exact biological interpretation in terms of single neurons since single neuronal spikes are not persistent and furthermore a single neuron cannot hold obtained spikes for later. Such a property

⋆ This work is supported by the Polish Minister of Scientific Research and Higher Education grant N N201 385234 (2008-2010).

could be however attributed to larger ensembles of neurons, particularly those having recurrent connections. In such case the local excitation can be preserved via loopback connections (in a way, stored). In this paper we show, that a grid of spiking neurons with short range excitation, long range inhibition stimulated via small amount of random noise converges to a homeostatic state, in which well delineated, persistent activity blobs emerge. Furthermore these blobs are able to travel significant distances across the domain. The total number of blobs is preserved (though some blobs vanish, and new are born) resembling the charge flow of the *spike flow model*.

2 The model

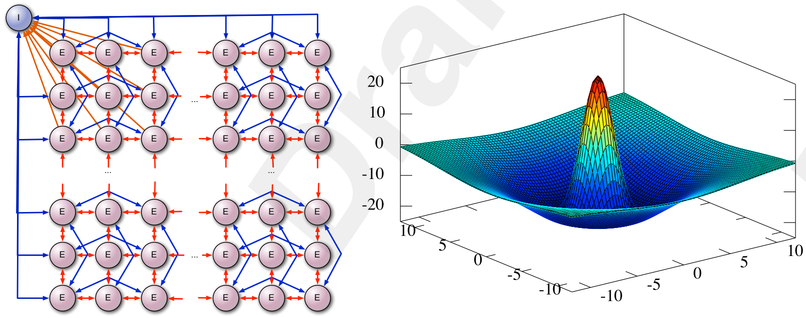

Fig. 1. The model consists of a grid of excitatory neurons connected to one inhibitory neuron, which has feedback inhibitory connections. Excitatory neurons excite their nearest neighbors but inhibit those at larger distances as $50 \cdot (e^{-d^2} - 0.5 \cdot e^{-0.5 \cdot d^2})$ (left) where $d = 0.5\sqrt{dx^2 + dy^2}$. The grid step is assumed to be 1.

The model consists of a grid of Eugene M. Izhikevich phenomenological simple neurons [8,9,10] governed by

$$\begin{cases} v' = & 0.04v^2 + 5v + 140 - u + I \\ u' = & a(bv - u) \end{cases} \tag{1}$$

the parameters are set as in the sample Matlab program in section IV of [8]. The connectivity is shown on figure 1. Neurons are organized into grid (torus topology), connected locally with mexican hat like weights computed from $50 \cdot (e^{-d^2} - 0.5 \cdot e^{-0.5 \cdot d^2})$ where $d = 0.5\sqrt{dx^2 + dy^2}$ is the grid distance (grid step is assumed to be 1). The inhibitory to excitatory weight was -20, while the excitatory to inhibitory weight was $\frac{10}{\#\text{ neurons}}$. The program was implemented in Matlab in a fashion similar to that of original E. Izhikevich script. Various sizes of the model were simulated, here we present figures obtained from 50x50 (1s - 1000 steps) and 80x80 (5s - 5000 steps) simulations.

In order to analyze the behavior of the blobs a following methodology was used:

- The voltage field was thresholded at fairly high value $v = -30$
- The resulting bitmap was cleaned with Matlabs `bwmorph`[1] function (single isolated pixels were removed)
- The bitmap was decomposed with Matlabs `bwlabel` funtion into individual blobs.
- Each blob was attributed a blob center in its center of mass.
- The list of existing blobs was looked up in search for nearest blobs to those found in current iteration.
- If previously existing blob was found within the distance of 3 units from any newly found, then the new one was resolved as the existing one that must have moved from previous time step. In that case the center of the existing blob gets updated (simple movement tracking).
- If there were no previously existing blobs within the range of 3 units, the blob was pronounced a new.
- After that step if some blob was neither updated nor created, it was removed from the list.

3 The spike flow model

The spike flow model has been introduced in [2,3] to comprehend with scale-free connectivity that has been found in some functional fMRI based networks [6,7] (none of the preexisting scale-free networks model was suitable to explain these phenomena).

The model consists of nodes $\sigma_i,\ i = 1 \ldots N$. Each node's state is described by a natural number from some fixed interval $[0, M_i]$. The network is built on a complete graph in that there is a connection between each pair of neurons $\sigma_i, \sigma_j,\ i \neq j$, carrying a real-valued weight $w_{ij} \in \mathbb{R}$ satisfying the usual symmetry condition $w_{ij} = w_{ji}$, moreover $w_{ii} := 0$. The values of w_{ij} are drawn independently from the standard Gaussian distribution $\mathcal{N}(0, 1)$ and are assumed to remain fixed in the course of the network dynamics. The model is equipped with the Hamiltonian of the form:

$$\mathcal{H}(\bar{\sigma}) := \frac{1}{2} \sum_{i \neq j} w_{ij} |\sigma_i - \sigma_j| \tag{2}$$

if $0 \leq \sigma_i \leq M_i,\ i = 1, \ldots, N$, and $\mathcal{H}(\bar{\sigma}) = +\infty$ in the other case. Here $\bar{\sigma}$ denotes of the state of the whole system. The dynamics of the network is defined as follows: at each step we randomly choose a pair of "neurons" (units) $(\sigma_i, \sigma_j),\ i \neq j$, and denote by $\bar{\sigma}^*$ the network configuration resulting from the original configuration $\bar{\sigma}$ by decreasing σ_i by one and increasing σ_j by one, that is to say by

[1] `bwmorph` and `bwlabel` are supplied with the image processing toolbox.

letting a unit charge transfer from σ_i *to* σ_j, whenever $\sigma_i > 0$ and $\sigma_j < M_j$. Next, if $\mathcal{H}(\bar{\sigma}^*) \leq \mathcal{H}(\bar{\sigma})$ we accept $\bar{\sigma}^*$ as the new configuration of the network whereas if $\mathcal{H}(\bar{\sigma}*) > \mathcal{H}(\bar{\sigma})$ we accept the new configuration $\bar{\sigma}^*$ with probability $\exp(-\beta[\mathcal{H}(\bar{\sigma}^*) - \mathcal{H}(\bar{\sigma})])$, $\beta > 0$, and reject it keeping the original configuration $\bar{\sigma}$ otherwise, with $\beta > 0$ standing for an extra parameter of the dynamics, in the sequel referred to as the inverse temperature conforming to the usual language of statistical mechanics.

The model results in a scale-free charge transfer graph with exponent $\gamma = 2$ (in agreement with empirical data), where the weight of each edge corresponds to the frequency of charge exchange events that were conducted along that edge (see [3] for details). The weak point of that model (though quite universal from mathematical point of view) is that it did not have an exact interpretation in terms of neurobiology, since it is not clear what the computational units and charge quants correspond to. The present paper is aimed to provide a direct[2] link between biologically feasible spiking networks and the *spike flow model*.

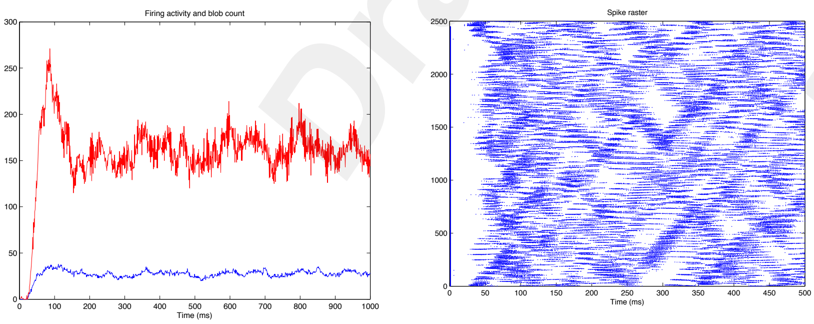

Fig. 2. Firing activity and the emergence of blobs on 50x50 domain. It takes some time before the model ignites the homeostatic blob dynamics (and it depends on the initial conditions). Nevertheless once the blobs emerge, they are persistent and firing activity levels off at some medium magnitude. The right figure shows the spike raster.

4 Results

At the beginning of the simulation the system fires single spikes in a unorganized manner, reflecting the random stimulation. At some point more spikes appear to synchronize. Eventually a number of activity blobs emerge and start to move across the domain. Soon after the initial activity jump (see figure 2 left), the system levels off in a homeostatic state in which the blobs are persistently emerging and moving (see figure 3)

[2] There are also other possible spike flow model interpretations, in terms of times spent by units in a certain dynamic attractor and so on. In this paper however, we show a rather straightforward interpretation.

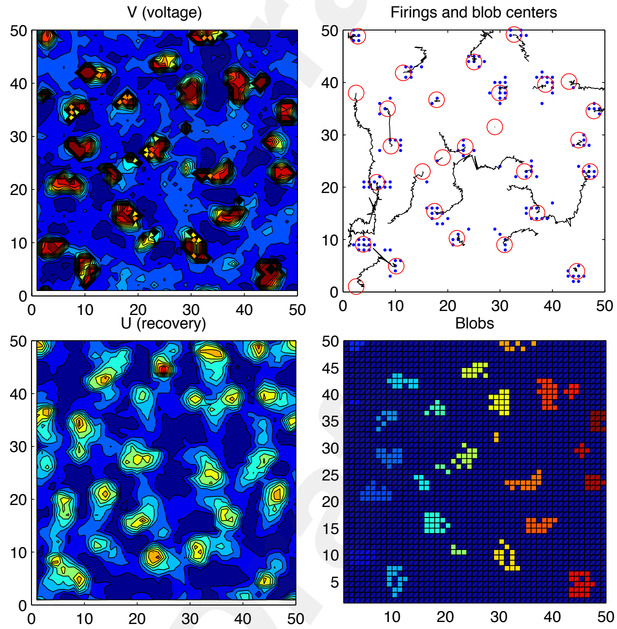

Fig. 3. 50x50 neuron domain. The voltage v is plotted on the top left subfigure. Bottom left shows the recovery variable u. Bottom right plot shows the thresholded voltage value divided into individual blobs. Each blob's center of a mass is shown with a circle on top right subfigure. The dots are spikes, the black lines are trajectories left by each blob as it moves. A real-time movie of the simulation of 80x80 domain is available `http://www.mat.umk.pl/~philip/ICAISC2009/80_realtime.mov` , and slowed down 8 times `http://www.mat.umk.pl/~philip/ICAISC2009/80_8xslower.mov`.

The rate at which the system arrives at the homeostatic regime depends somewhat on the initial conditions. It seems that the system requires some time to synchronize. Artificially firing all neurons at the first step leads to faster convergence[3]. Nevertheless once the blobs emerge, they stay forever[4], and so the initial conditions don't seem to be very important for blob features.

An important question addressed in this paper is whether the blobs satisfy the conditions which allow them to be considered as the charge packets exchanged in the spike flow model, that is:

- Is the average blob lifetime long?
- Do blobs manage to travel long distances?

[3] Artificial firing of all neurons is the initial condition used in this paper.
[4] Existence of blobs is very stable in the model with local mexican hat connectivity (presented here). However addition of random edges may introduce time periodicity (caused by desynchronization) and cause blobs to vanish and reappear and so on.

– Are blobs similar in sizes and activations they carry?

These questions will be answered in paragraphs below.

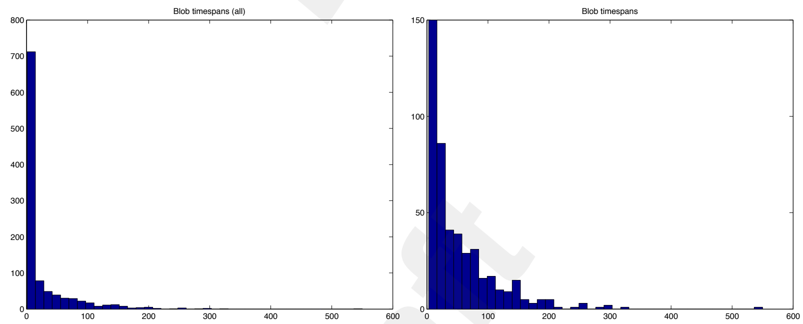

Fig. 4. The distribution of timespans among the blobs on 50x50 domain. As previously the left figure shows all timespans, while the right one only those which persisted for more than 2ms.

Lifespan The blobs are rather persistent. As seen in figure 4 for the 50x50 domain some blobs are able to survive nearly 600ms. The distribution however is concentrated on short living blobs. This is due to the properties of blob finding algorithm - many blobs are only pinpointed in a single time step. This happens frequently, whenever two or three random spikes appear nearby. Such random fluctuations do not give a rise to a "real" blob, but instead mess the statistics with false positives. Since due to short lifetime these false positive blobs do not move, we compute some statistics after throwing away blobs living less than some threshold (2-10 time steps). Median timespan of "true" blobs is between 40-70ms (depending on the size of the domain and time of simulation). The oldest blobs at 80x80 domain arrive near 1s life, which is exactly the timescale expected with the spike flow model.

Distances While alive, blobs move across the domain. It turns out their movement is not like a random walk in which they would constantly change the direction of movement and eventually drifted in brownian manner. Instead it seems like the blobs have true velocities which are changed rarely (see sample trajectories on figure 3 top right). The best blobs managed to travel nearly 400 distance units on 50x50 domain. With 80x80 domain some blobs traveled nearly 2000 units (with median near 100)! Furthermore these statistics can be diminished by the methodology - since the domain is a torus some blobs disappear at one side and reappear at the other. With the current algorithm such spots are treated separately, whereas in fact they are the same blob. Nevertheless, even with the imperfect statistics the conclusion is that blobs manage to travel significant distances.

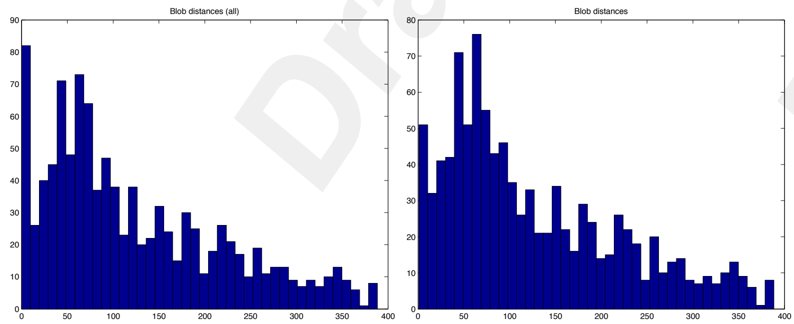

Fig. 5. The distribution of distances traveled by blobs on 50x50 domain during 4000ms simulation. The left figure shows all distances including those, traveled by flase positive blobs (which survive only one time step), the right one shows only those which persisted at least 2 ms.

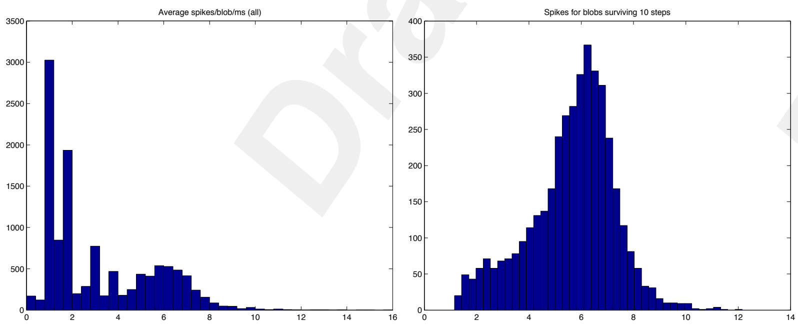

Fig. 6. The distribution of an average number of spikes firing within the blob per time step. After neglecting false positives (blobs that only emerged for very short time, in this case < 10) one obtains a fairly centered distribution. This shows that all persistent blobs are rather equally active.

Activities The figure 3 (bottom right) might suggest that the blobs are of various sizes and shapes. This is a rather false conviction, since the spots constantly change, at one frame the blob appears as large while a few steps later it is very small. To obtain a trustworthy statistics about the blobs activity a following method was used:

– Whenever a neuron spiked, an algorithm looked for a blob within a distance of 6 units from the spike
– If it found one, the spike was attributed as coming from that particular blob. In other case it was ignored.
– When the spot was at the end of its life, the total number of spikes it collected was divided by its lifetime, to obtain average spiking activity. These averages were saved to obtain a histogram in figure 6.

The results are shown in figure 6. Again the statistics are corrupted with false positive transient blobs. After throwing away any spot that survived less that 10 steps, one obtains a fairly well centered distribution with a majority of blobs having 5-7 spikes/ms. This shows that in fact most of the blobs are much alike, and carry the same amount of "activity"

5 Conclusions

The presented model is aimed at bridging biologically plausible dynamical spiking neural networks with the *spike flow model* which itself is aimed at showing where the scale-free connectivity in functional brain networks might come from. The persistent activity blobs described here seem to be right candidates for the charge units that are being exchanged in the spike flow model. As shown by simulations, the blobs are persistent, travel significant distances, operate on the right timescale and on average carry the same amount of "activity". The "neurons" of the *spike flow model* can in this context be interpreted as subsets of the domain. In this simple case the domain in 2d (resembling the cortex), but such blobs should also appear with higher dimensionality. In particular the long range myelinated cortico-cortical connections can form wormholes that teleport a blob from one cortical area to another, giving them more freedom (the spike flow model in the original setup is a mean field model, but many of its properties remain valid when it is submerged in rich enough topology).

Acknowledgments. The author gratefully acknowledges the support from the Polish Minister of Scientific Research and Higher Education grant N N201 385234 (2008-2010). The author also appreciates fruitful collaboration with Dr Tomasz Schreiber.

References

1. Izhikevich, E.M.: Polychronization: Computation with spikes. Neural Comput. **18**(2) (2006) 245–282
2. Piękniewski, F., Schreiber, T.: Emergence of scale-free spike flow graphs in recurrent neural networks. In: Proc. IEEE FOCI. (2007) 357–362
3. Piękniewski, F., Schreiber, T.: Spontaneous scale-free structure of spike flow graphs in recurrent neural networks. Neural Networks **21**(10) (2008) 1530 – 1536
4. Piękniewski, F.: Emergence of scale-free graphs in dynamical spiking neural networks. In: Proc. IEEE International Joint Conference on Neural Networks, Orlando, Florida, USA, IEEE Press (August 2007) 755–759
5. Piekniewski, F.: Robustness of power laws in degree distributions for spiking neural networks. In: Proc. IEEE 2009 International Joint Conference on Neural Networks (IJCNN), Atlanta, Georgia, USA, IEEE Press (June 2009) 2541–2546
6. Sporns, O., Chialvo, D.R., Kaiser, M., Hilgetag, C.C.: Organization, development and function of complex brain networks. Trends Cogn Sci **8**(9) (September 2004) 418–425
7. Eguíluz, V.M., Chialvo, D.R., Cecchi, G.A., Baliki, M., Apkarian, A.V.: Scale-free brain functional networks. Phys Rev Lett **94**(1) (January 2005)
8. Izhikevich, E.M.: Simple model of spiking neurons. IEEE Transactions on Neural Networks (14) (2003) 1569– 1572
9. Izhikevich, E.M.: Which model to use for cortical spiking neurons? Neural Networks, IEEE Transactions on **15**(5) (Sept. 2004) 1063–1070
10. Izhikevich, E.M.: Dynamical systems in Neuroscience: The Geometry of Excitability and Bursting. MIT Press., Boston (2006)